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Research Article

Optimization of Inventory Management Logistic Model of the Machine-Building Enterprises

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Abstract

The aim of this study is to develop the inventory management model based on Economic Order Quantity model using asymptotic perturbation methods. The simple asymptotic formulas for the "perturbed" order quantity has been obtained when cost per order, storage cost and product demand change slightly. As the results show, the total costs, which correspond to "perturbed" order quantities, are less than ones at economic order quantity. Decrease of logistics costs can improve the market competitiveness of the machine-building enterprises' products. Modeling the nature of cost increase and demand fluctuation using asymptotic formulas the machine-building enterprises will be able to make prompt adjustments to optimize logistics processes.

Keywords: Logistic model, asymptotic methods, small parameter, asymptotic expansions.

JEL Codes : C59, M11, M20

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1. Introduction

The increase in economic interdependence at the level of states, regions and specific companies is an indicator of modern social and economic development of the world which includes Ukraine and entities of entrepreneurial activity. The integration of the Ukrainian economy into the global economic system causes a growth of competition from the high-production international companies. The efficiency of enterprises' activity in a competitive environment is achieved using modern principles, approaches and management methods in various fields.

Machine-building industry is one of the most important sectors of the modern economy which greatly affects the production, efficiency and progress in almost all fields of human activity. Machine-building in Ukraine is presented by metallurgy, mining, energy, ship-building, aircraft-building and automobile industry, manufacturing of machines and devices for chemical, light and food industries, agricultural machine-building, road-building, manufacturing of machine tools, machinery and equipment for the armed forces, etc.

The current state of Ukrainian economy is characterized by significant deterioration of economic indicators reflecting the enterprise performance, especially in machine-building. It is known that expenses for the material resources make up half the cost of engineering products. So, one of the key positions to ensure the competitiveness of machine-building enterprises is a supply activity optimization through the inventory management system improvement.

Econometric modelling of logistic systems provides for application of modern mathematical tools for optimization, especially towards inventory management. The models that are applied to decision-making on enterprise inventory management have many restrictions. Deviations of simplified mathematical model from the real logistic system may be different. In case when the original system parameters do not change significantly, deviations in the results of modelling may be small in the whole field of options. To analyze the "perturbed" system the asymptotic methods can be used. These methods make it possible to find a solution of the problem provided a small range of variation of system parameters.

The aim of this study is further development of inventory management model using asymptotic methods. In this paper an economic order quantity model has been developed using the perturbation techniques when cost per order, storage one and demand for products change slightly. The simple asymptotic formulas for the "perturbed" order quantity can help to estimate its deviation from the economic one, to calculate the total cost and to adjust inventory management logistic subsystem.

This study is organized as follows: Section 2 discuss the literature review. Section 3 presents the essence of asymptotic methods. Section 4 presents the asymptotic approach to Economic Order Quantity model provided rising both per order and storage costs. Section 5 presents the asymptotic approach to Economic Order Quantity model provided demand fluctuations. Finally, the references appear at this study's conclusion.

2. Literature Review

It should be noted that there is a vast amount of literature on inventory management. Pentico et al. (2011), Lukinskiy (2007) review deterministic models and methods that help managers in solving specific logistical problems. The most common analytical model of applied logistic is EOQ - model (Economic Order Quantity). Applying of this model is limited to several assumptions, including the constancy of cost per order and storage one. In practice, as a rule, these conditions are not met. Therefore, the researchers are interested in the impact of variation of the model input parameters on the final result.

In order to make EOQ - model more applicable in practice factors such as imperfect quality items, delay of payment, deteriorating inventory have been considered by many researchers recently.

Jaggi et al. (2013) developed an EOQ based inventory model for imperfect quantity items to determine the optimal ordering policies of a retailer under permissible delay in payments with allowable shortages. An inventory model with shortage and exponential demand rate under permissible delay in payments was presented in the work of Tripathi (2012). Tripathi et al. (2013) developed a model for optimal order policy for time-dependent deteriorating items in response to temporary price discount linked to order quantity. In a recent work of Tripathi et al. (2015) the inventory model for a linear time-dependent demand rate has been developed when holding cost is proportional to time and a supplier provides a permissible delay in payments. Tyagi (2014) studies an inventory model in which the inventory is depleted not only by declining pattern of demand but also by Weibull distributed deterioration where holding cost per unit time is considered a discretely variable. In the study of Sinha (2014) an Entropic order quantity model has been proposed when all the items received are not of good quality and demand is price sensitive. Shukla et al. (2015) studies economic ordering policies with linearly time-dependent demand rate under trade credits. Vijayashree et al. (2015) developed EOQ models for perishable products which consider continuous deterioration of a utility product and introduce an exponential penalty cost and linear penalty cost function.

Some of the researchers (Eynan et al., 2007), (Wang, 2010), (Yan et al., 2008) consider variations in the real world situations (inflation, sudden rise and fall in the economy and so on) and present inventory models in the uncertain environments. In the work of Yousefli et al. (2012) economic order quantity problem has been developed in the stochastic environment in which the storage cost, setup cost and inventory space have probability distribution functions. In the study of Ritha et al. (2013) the demand and associated costs are taken as fuzzy variables and in order to determine the optimal inventory policies the Yager's ranking methods for fuzzy numbers are used. Dutta et al. (2012) use triangular and trapezoidal fuzzy number in building inventory models for determining the optimal order quantity and the optimal cost. Kaur et al. (2014) determine the optimal cost and an optimum order quantity of inventory by taking certain non-deterministic parameters as triangular intuitionistic fuzzy numbers.

Although the optimization methods can solve inventory problems efficiently because of their complexity they are often incomprehensible to the practitioners such as managers. Both for researchers and for managers the analytical models, that describe the object behavior by understandable formulas, are more convenient to use.

3. Essence of Asymptotic Methods

Most of the mathematical models of real processes have a number of significant features that do not allow researchers to obtain accurate analytical solutions. To solve these problems, researchers have to use different approximate methods. Among the approximate analytical methods, the asymptotic perturbation methods in a small parameter are the most important.

The essence of asymptotic methods is that the solution to the problem is presented as an expansion in a series in powers of a small parameter that occurs naturally or introduced artificially for convenience. To study applied problem the solution form in a series is assumed and the appropriate asymptotic sequence is selected. The simplest and commonly used in practice is the sequence of whole powers of the small parameter in particular ε^n. An approximate solution has an asymptotic nature in the sense that it is close to the exact solution not by increasing the number of expansion terms N but when N is fixed and small parameter tends to zero, that is:

$$f(x,\varepsilon)=\sum_{n=0}^{N} a_n(x)\delta_n(\varepsilon)+o(\delta_N(\varepsilon)) \quad \text{when } \varepsilon \to 0, \tag{1}$$

where $\delta_n(\varepsilon)$ is an asymptotic sequence.

In practice the solution is presented by the first few terms of asymptotic expansion, the number of which usually is not more than two.

Detailed description and practical application of the asymptotic methods and perturbation techniques are given in the works of Nayfeh (1981), Andrianov et al. (1994), Gristchak (2009).

The advantages of asymptotic methods are the simplicity and accuracy due to location. The basic idea is that a simplified solution of the problem is in the vicinity of a boundary condition and this solution is more accurate the smaller this vicinity. Moreover, asymptotic methods make it possible to use previously obtained analytical solutions of applied problems for solving similar but more complicated ones by setting connections between them.

Asymptotic methods are widely used in mechanics of solid structures (Steele, 1989), (Koiter et al., 1994), (Gristchak et al., 2003), biomechanics (Stein et al., 2003), mathematical analysis, differential equations (Geer et al., 1989), (Gristchak et al., 1995), but the issue of their use in economic and particular in the logistic models is still open and requires advanced study.

4. Asymptotic Approach to Economic Order Quantity Model Provided Rising per Order and Storage Costs

The practice of enterprise inventory management provides for the use of one of the most common models that is known as economic order quantity model (EOQ - model) or Wilson formula. Wilson formula for estimation the economic order quantity may take the following forms (Lukinskiy, 2007):

$$q_{opt}=\sqrt{\frac{2C_0 S}{h}} \quad \text{or} \quad q_{opt}=\sqrt{\frac{C_0 S}{\alpha \cdot k}}, \tag{2}$$

where q_{opt} is economic order quantity; C_0 is cost per order, S is the demand quantity for a certain period of time; h is storage cost for a certain period of time, α is storage cost per unit for a certain period taking into account the occupied area; k is a factor for the outline dimensions of the unit.

One of the main parts of cost per order is shipping cost which because of rising fuel prices increases constantly. Assuming that over the certain period of time (for instance, each month) the cost per order increases by *i*%, then after *n* periods it will reach $C_0 \cdot \left(1+\frac{i\%}{100\%}\right)^n$.

Taking ratio $\varepsilon=\frac{i\%}{100\%}$ ($\varepsilon \ll 1$) as a small parameter, we obtain the cost per order in the following form $C_0 \cdot (1+\varepsilon)^n$. Since the parameter ε is small we can assume that the deviation from the initial value C_0 is not significant, and condition of cost per order constancy is satisfied.

The "perturbed" optimal order quantity q^*_{opt} can be represented as an asymptotic expansion in the artificially introduced small parameter ε:

$$q^*_{opt} = q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots . \tag{3}$$

We neglect here the terms of higher order then ε^2.

Then formulas for the order quantity take the forms (4) or (5):

$$q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots = \sqrt{\frac{2C_0 (1+\varepsilon)^n S}{h}}; \tag{4}$$

$$q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots = \sqrt{\frac{C_0 (1+\varepsilon)^n S}{\alpha k}}, \tag{5}$$

where ε is perturbation parameter.

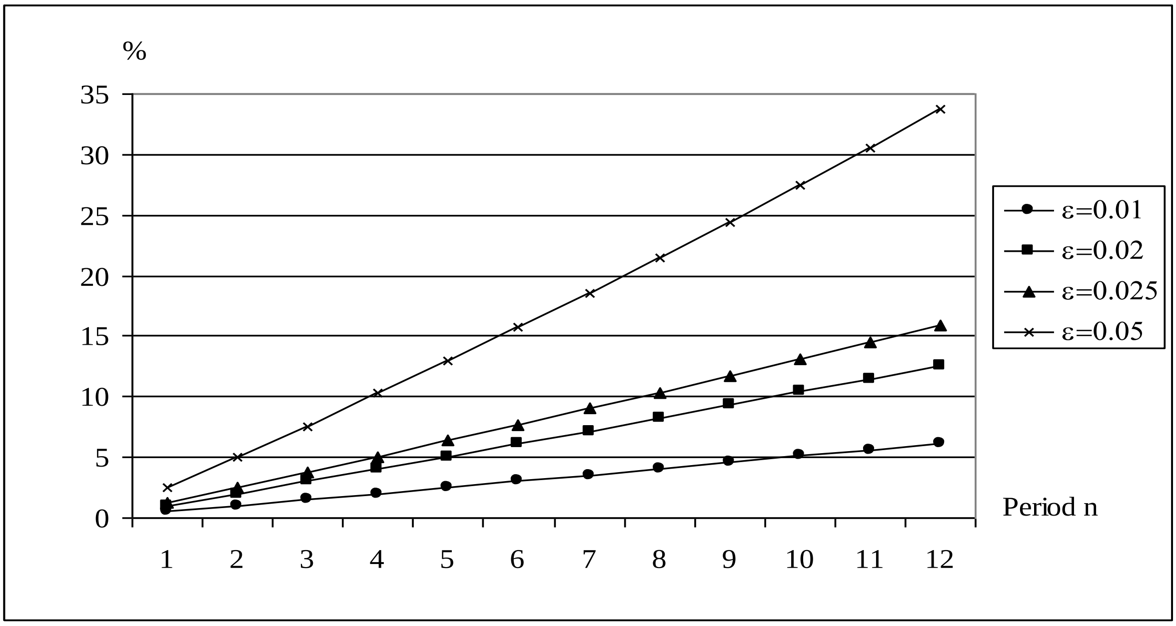
Expanding function $(1+\varepsilon)^{\frac{n}{2}}$ in a Taylor series, we obtain the asymptotic formulas for the "perturbed" order quantity q^*_{opt} in the forms

$$q^*_{opt} = \sqrt{\frac{2C_0 S}{h}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right); \tag{6}$$

$$q^*_{opt} = \sqrt{\frac{C_0 S}{\alpha k}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right). \tag{7}$$

As we can see from (6)-(7), "perturbed" order quantity differs from Wilson formula (2) for the multiplier $\left(1 + \frac{n}{2} \cdot \varepsilon + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right)$.

Figure 1: "Perturbed" Order Quantity Deviation from Wilson Formula



Source: Calculated by the authors.

Percentage of “perturbed” order quantity deviation from the economic one is given in Figure 1.

As can be seen in Figure 1, the gradual increase of cost per order by 1 % ($\varepsilon = 0.01$) leads to the increase of the order quantity by 3.0 % and 6.2 % in the periods 6 and 12 respectively. If the cost per order gradually increase, for example, by 5 % ($\varepsilon = 0.05$) each period, the order quantity will increase by 15.8 % in period 6.

Comparing the total costs TC , which correspond to economic (2) and “perturbed” (6)-(7) order quantities, we can see that provided slight increase of cost per order the total cost reaches the minimum at “perturbed” order quantity:

$$TC\left(q_{opt}\right) = \frac{C_0(1+\varepsilon)^n S}{q_{opt}} + \frac{h q_{opt}}{2} = \sqrt{\frac{C_0 S h}{2}} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-1)}{2} \cdot \varepsilon^2\right); \quad (8)$$

$$TC^*\left(q^*_{opt}\right) = \frac{C_0(1+\varepsilon)^n S}{q^*_{opt}} + \frac{h q^*_{opt}}{2} = \sqrt{\frac{C_0 S h}{2}} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-2)}{4} \cdot \varepsilon^2\right); \quad (9)$$

$$TC\left(q_{opt}\right) = \frac{C_0(1+\varepsilon)^n S}{q_{opt}} + \alpha k q_{opt} = \sqrt{C_0 S \alpha k} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-1)}{2} \cdot \varepsilon^2\right); \quad (10)$$

$$TC^*\left(q^*_{opt}\right) = \frac{C_0(1+\varepsilon)^n S}{q^*_{opt}} + \alpha k q^*_{opt} = \sqrt{C_0 S \alpha k} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-2)}{4} \cdot \varepsilon^2\right). \quad (11)$$

In practice, not only the cost per order but also the storage cost rises as a result of electricity and utilities prices increase. Assume that the storage cost increases each period of time by *j*%. Similarly, taking value $\beta = \frac{j\%}{100\%}$ ($\beta << 1$) as a small parameter, we obtain a dependence of storage cost in the forms $\alpha \cdot (1+\beta)^m$ or $h \cdot (1+\beta)^m$.

Changing of shipping cost, which is one of the main components in the structure of cost per order, and the rising of the utilities prices that increases accordingly storage cost, often occur at different periods of time. Thus, due to the gradual devaluation of the Ukrainian currency the prices of fuel and lubricants and ones for the imported car spare parts increase and this is reflected in transport fares. At the same time prices for electricity, heating and water supply increase significantly every six months. So, it is reasonable to consider various combinations of parameter values *n* and *m*, ε and β in EOQ-model taking into account that an interval of storage cost change *m* is smaller relative to the one of order cost change *n*, but parameter β, which characterizes growth of storage cost, exceeds the value of parameter ε, that characterizes rising cost per order. We can consider that the change in storage cost happens with a delay in comparison with change in the cost per order putting $m = \left[\frac{n}{4}\right]$, $m = \left[\frac{n}{6}\right]$ etc., where [ ] denotes a whole number. Because of the smallness of ε and β parameters, we can assume that the deviations from the initial values C_0 and h or α are small and restrictions, that take place in the model (2), are almost not violated. Under these conditions the modified formula for order quantity takes the forms (12) or (13):

$$q^*_{opt} = \sqrt{\frac{2C_0S}{h}} \cdot \sqrt{\frac{(1+\varepsilon)^n}{(1+\beta)^m}}\,; \tag{12}$$

$$q^*_{opt} = \sqrt{\frac{C_0S}{\alpha k}} \cdot \sqrt{\frac{(1+\varepsilon)^n}{(1+\beta)^m}}\,, \tag{13}$$

where ε and β are perturbation parameters.

Formulas (12)-(13) are not suitable for the use in practice because it complicates the calculation of the order quantity when the period numbers *n* and *m* increase.

Representing q^*_{opt} as an asymptotic expansion in two small parameters ε and β, we neglect the terms of the order ε^3, β^3, $\varepsilon^2\beta$, $\varepsilon\beta^2$ and higher:

$$q^*_{opt} = q_0 + q_1 \cdot \varepsilon + q_2 \cdot \beta + q_3 \cdot \varepsilon^2 + q_4 \cdot \varepsilon \cdot \beta + q_5 \cdot \beta^2 + \ldots \tag{14}$$

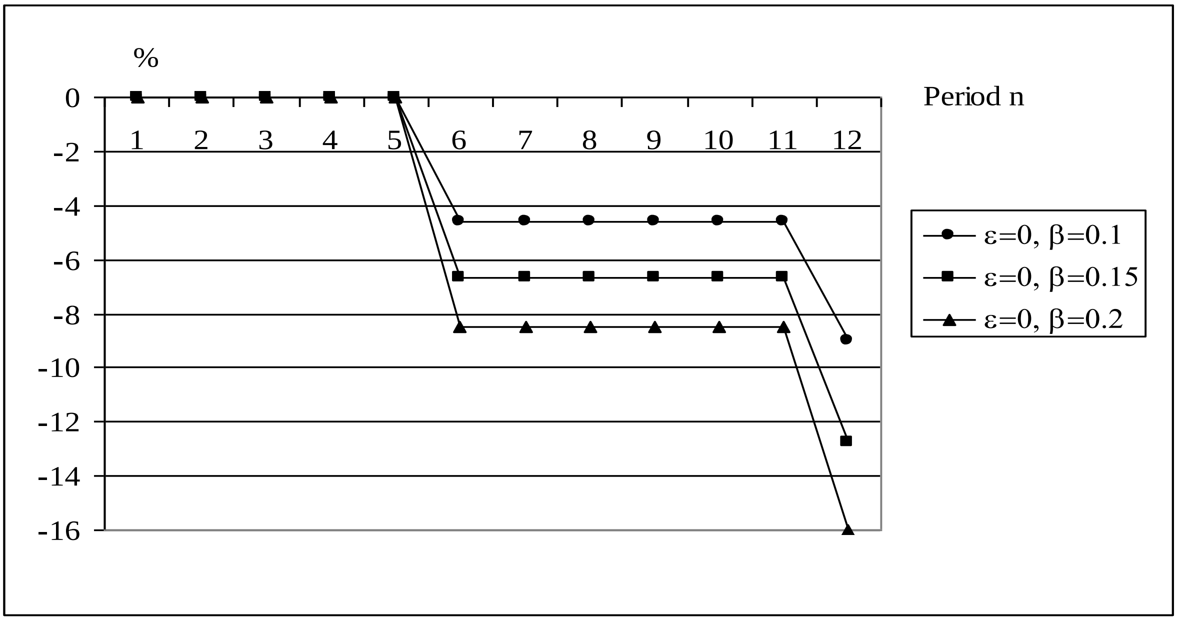
Expanding functions $(1+\varepsilon)^{\frac{n}{2}}$ and $(1+\beta)^{-\left(\frac{m}{2}\right)}$ in a Taylor series, we obtain the asymptotic formulas in two parameters ε and β for the "perturbed" order quantity in the forms (15) or (16):

$$q^*_{opt} = \sqrt{\frac{2C_0S}{h}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon - \frac{m}{2}\beta + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2 - \frac{m \cdot n}{4} \varepsilon \cdot \beta + \frac{m \cdot (m+2)}{8} \beta^2\right); \tag{15}$$

$$q^*_{opt} = \sqrt{\frac{C_0S}{\alpha k}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon - \frac{m}{2}\beta + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2 - \frac{m \cdot n}{4} \varepsilon \cdot \beta + \frac{m \cdot (m+2)}{8} \beta^2\right). \tag{16}$$

To analyze the order quantity sensitivity to the change of cost per order and storage cost, the calculations of ratio of "perturbed" order quantity to the optimal one (2) have been done at different values of parameters ε, β and different periods *n* and *m*.

Figure 2: "Perturbed" Order Quantity Deviation from the Economic Order Quantity Provided Cost per Order is Fixed, ($\varepsilon = 0$, $m = \left[\frac{n}{6}\right]$)



Source: Calculated by the authors.

Percentage of "perturbed" order quantities deviation from the economic order quantities (2), provided the cost per order is fixed, is given in Figure 2.

Ratio of "perturbed" order quantities (15)-(16) to the economic ones (2) and percentage of deviation provided that both cost per order and storage cost vary slightly are given in Table 1.

Table 1: Ratio of "perturbed" order quantity to the economic order quantity provided the change of both cost per order and storage cost, $m = \left[\frac{n}{6}\right]$

Period		$\varepsilon = 0.01, \beta = 0.1$		$\varepsilon = 0.01, \beta = 0.2$		$\varepsilon = 0.02, \beta = 0.1$		$\varepsilon = 0.02, \beta = 0.2$	
n	m	$\frac{q^*_{opt}}{q_{opt}}$	%	$\frac{q^*_{opt}}{q_{opt}}$	%	$\frac{q^*_{opt}}{q_{opt}}$	%	$\frac{q^*_{opt}}{q_{opt}}$	%
1	0	1.005	+0.5	1.005	+0.5	1.010	+1.0	1.010	+1.0
2	0	1.010	+1.0	1.010	+1.0	1.020	+2.0	1.020	+2.0
3	0	1.015	+1.5	1.015	+1.5	1.030	+3.0	1.030	+3.0
4	0	1.020	+2.0	1.020	+2.0	1.040	+4.0	1.040	+4.0
5	0	1.025	+2.5	1.025	+2.5	1.051	+5.1	1.051	+5.1
6	1	0.983	–1.7	0.942	–5.8	1.012	+1.2	0.970	–3.0
7	1	0.987	–1.3	0.947	–5.3	1.022	+2.2	0.980	–2.0
8	1	0.992	–0.8	0.952	–4.8	1.032	+3.2	0.989	–1.1
9	1	0.997	–0.3	0.956	–4.4	1.042	+4.2	0.999	–0.1
10	1	1.002	+0.2	0.961	–3.9	1.053	+5.3	1.009	+0.9
11	1	1.007	+0.7	0.966	–3.4	1.063	+6.3	1.019	+1.9
12	2	0.966	–3.4	0.890	–11.0	1.024	+2.4	0.942	–5.8

Source: Calculated by the authors.

Table 2: Ratio of "perturbed" order quantity to the economic one depending on *m* parameter change

Period		$\varepsilon = 0.01$, $\beta = 0.1$	$\varepsilon = 0.01$, $\beta = 0.2$	Period		$\varepsilon = 0.01$, $\beta = 0.1$	$\varepsilon = 0.01$, $\beta = 0.2$
n	m	$\frac{q^*_{opt}}{q_{opt}}$	$\frac{q^*_{opt}}{q_{opt}}$	n	m	$\frac{q^*_{opt}}{q_{opt}}$	$\frac{q^*_{opt}}{q_{opt}}$
1	0	1.005	1.005	1	0	1.005	1.005
2	0	1.010	1.010	2	0	1.010	1.010
3	0	1.015	1.015	3	0	1.015	1.015
4	1	0.973	0.933	4	0	1.020	1.020
5	1	0.978	0.938	5	0	1.025	1.025
6	1	0.983	0.942	6	1	0.983	0.942
7	2	0.942	0.868	7	1	0.987	0.947
8	2	0.947	0.873	8	1	0.992	0.952
9	2	0.951	0.877	9	1	0.997	0.956
10	3	0.912	0.811	10	1	1.002	0.961
11	3	0.917	0.815	11	1	1.007	0.966
12	3	0.921	0.819	12	2	0.966	0.890

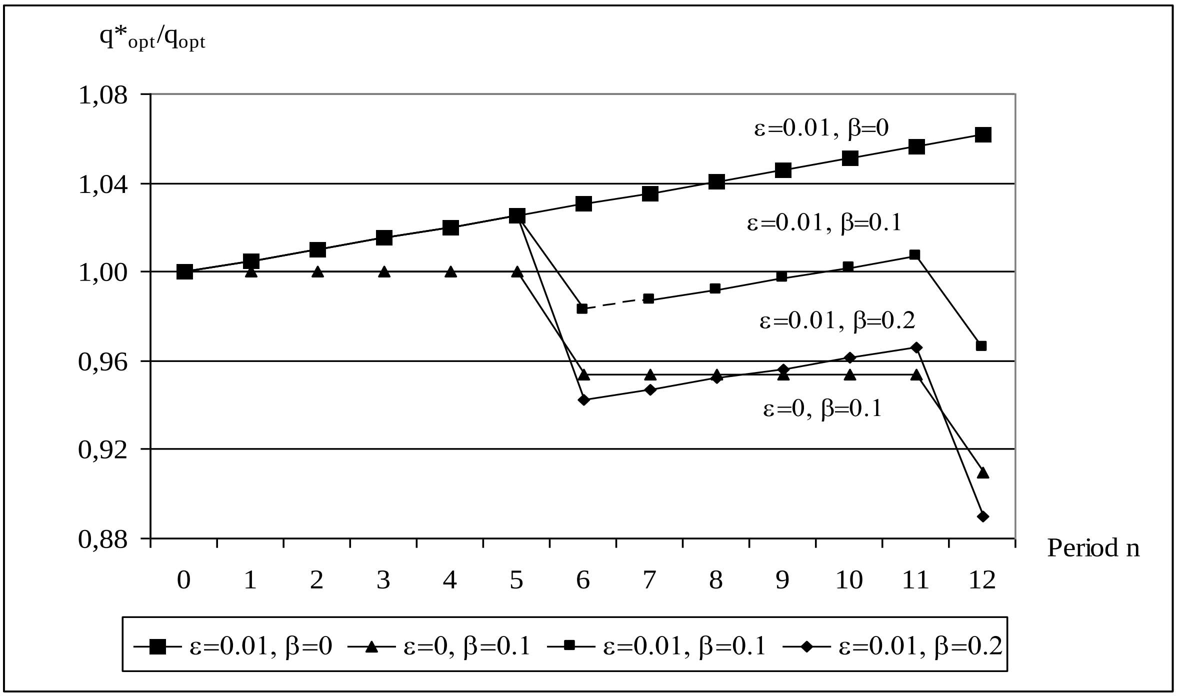
Source: Calculated by the authors.

As we can see, the increase of the storage tariff rate every three periods causes significant abrupt decrease of the "perturbed" order quantity compared to the economic one. Thus, when $\varepsilon = 0.01$ and $\beta = 0.1$ in period 4 the deviation of order quantity is –2.7 %, in period 7 it is –5.8 % and in period 10 the deviation is –8.8 %. In case the enterprise purchases products per large scale batches such a deviation in absolute units is significant.

Figure 3 clearly demonstrates the nature of the "perturbed" order quantity deviation from the economic order quantity under different conditions of gradual increase of per order and storage costs.

By modeling the nature of the changes in cost per order and utility rates in short term period, the entrepreneur can make appropriate adjustments to the purchase organization of the company by determining the order quantity with asymptotic formulas (15)-(16).

Figure 3: Deviation of "Perturbed" Order Quantity from the Economic One Depending on Parameters ε, β and n ($m = \left[\frac{n}{6}\right]$)



Source: Calculated by the authors.

Total costs TC provided slight increase of both cost per order and storage one at economic (2) and "perturbed" (15)-(16) order quantities take forms:

$$TC\left(q_{opt}\right)=\frac{C_0(1+\varepsilon)^n S}{q_{opt}}+h\,(1+\beta)^m\,\frac{q_{opt}}{2}= \\ =\sqrt{\frac{C_0 S\,h}{2}}\left(2+n\cdot\varepsilon+m\cdot\beta+\frac{n\cdot(n-1)}{2}\cdot\varepsilon^2+\frac{m\cdot(m-1)}{2}\cdot\beta^2\right); \quad (17)$$

$$TC^*\left(q^*{}_{opt}\right)=\frac{C_0(1+\varepsilon)^n S}{q^*{}_{opt}}+h\,(1+\beta)^m\,\frac{q^*{}_{opt}}{2}=\sqrt{\frac{C_0 S\,h}{2}}\times \\ \times\left(2+n\cdot\varepsilon+m\cdot\beta+\frac{n\cdot(n-2)}{4}\cdot\varepsilon^2+\frac{m\cdot(m-2)}{4}\cdot\beta^2+\frac{m\cdot n}{2}\cdot\varepsilon\cdot\beta\right); \quad (18)$$

$$TC\left(q_{opt}\right)=\frac{C_0(1+\varepsilon)^n S}{q_{opt}}+\alpha\,(1+\beta)^m\,k\,q_{opt}= \\ =\sqrt{C_0 S\alpha\,k}\left(2+n\cdot\varepsilon+m\cdot\beta+\frac{n\cdot(n-1)}{2}\cdot\varepsilon^2+\frac{m\cdot(m-1)}{2}\cdot\beta^2\right); \quad (19)$$

$$TC^*\left(q^*{}_{opt}\right)=\frac{C_0(1+\varepsilon)^n S}{q^*{}_{opt}}+\alpha\,(1+\beta)^m\,k\,q^*{}_{opt}=\sqrt{C_0 S\alpha\,k}\times \\ \times\left(2+n\cdot\varepsilon+m\cdot\beta+\frac{n\cdot(n-2)}{4}\cdot\varepsilon^2+\frac{m\cdot(m-2)}{4}\cdot\beta^2+\frac{m\cdot n}{2}\cdot\varepsilon\cdot\beta\right). \quad (20)$$

After simple transformations (17)-(20) we can see that the total cost TC, which corresponds to "perturbed" order quantity (22)-(24), is less than one at economic order quantity (21)-(23):

$$TC\left(q_{opt}\right)=\sqrt{\frac{C_0 S h}{2}}\times\left(\Omega+\frac{1}{2}(n\cdot\varepsilon-m\cdot\beta)^2\right); \tag{21}$$

$$TC^*\left(q^*{}_{opt}\right)=\sqrt{\frac{C_0 S h}{2}}\times\left(\Omega+\frac{1}{4}(n\cdot\varepsilon-m\cdot\beta)^2\right); \tag{22}$$

$$TC\left(q_{opt}\right)=\sqrt{C_0 S\alpha k}\times\left(\Omega+\frac{1}{2}(n\cdot\varepsilon-m\cdot\beta)^2\right); \tag{23}$$

$$TC^*\left(q^*{}_{opt}\right)=\sqrt{C_0 S\alpha k}\times\left(\Omega+\frac{1}{4}(n\cdot\varepsilon-m\cdot\beta)^2\right), \tag{24}$$

where $\Omega=\left(2+n\cdot\varepsilon+m\cdot\beta-\frac{n}{2}\cdot\varepsilon^2-\frac{m}{2}\cdot\beta^2+m\cdot n\cdot\varepsilon\cdot\beta\right)$.

For enterprises of the machine-building industry the inventory management is a key subsystem that affects their competitiveness on the current market. They can optimize their logistics costs for the purchase of parts and materials, taking into account the order and storage cost increase due to rising prices for fuel and lubricants, utilities prices and because of inflation. The developed model makes it possible to consider these changes.

JSC "Motor Sich" is the leading enterprise of Ukrainian machine-building. The main focus of its activities is the production of aircraft engines and consumer goods. Among the product range of consumer goods are agricultural machinery, petrol and electric saws, separators for milk, etc. We analyze the effect of per order and storage costs increase on the total cost when purchasing the engines DK 110-60 for JSC "Motor Sich" separators for milk. The parameter values are taken as: annual demand $S=10{,}000$ units, cost per order $C_0=1000\ UAH$, annual storage cost $h=48\ UAH$.

Table 3 presents the effect of per order and storage costs increase on the total costs, which correspond to economic and "perturbed" order quantities.

Table 3: Comparison of the total costs, UAH

Period		$\varepsilon=0.02,\ \beta=0$		$\varepsilon=0.02,\ \beta=0.1$		$\varepsilon=0.02,\ \beta=0.2$	
n	m	TC (17)	TC^* (18)	TC (17)	TC^* (18)	TC (17)	TC^* (18)
0	0	30984	30984	30984	30984	30984	30984
1	0	31294	31292	31294	31292	31294	31292
2	0	31610	31604	31610	31604	31610	31604
3	0	31932	31918	31932	31918	31932	31918
4	0	32260	32236	32260	32236	32260	32236
5	0	32595	32556	32595	32556	32595	32556
6	1	32936	32880	34485	34483	36034	36009
7	1	33283	33207	34832	34826	36381	36367
8	1	33636	33537	35185	35171	36734	36728
9	1	33995	33870	35545	35520	37094	37092
10	1	34361	34206	35910	35872	37459	37459

11	1	34733	34545	36282	36226	37831	37830
12	2	35111	34888	38364	38358	41927	41828

Source: Calculated by the authors.

As can be seen from Table 3, the total costs that correspond to the "perturbed" order quantity and have been calculated using formula (18), are slightly less than those have been calculated using formula (17). However, in scales of the total purchases of the machine-building enterprise cost reductions can be significant.

5. Asymptotic Approach to Economic Order Quantity Model Provided Demand Fluctuations

In practice besides variations in cost per order and storage cost demand S for the products undergoes slight fluctuations also. These fluctuations may be caused by intense market competition, seasonality of demand, etc.

We assume that cost per order takes the form $C_0(1+\varepsilon)^n$. Periodic changes in demand S can be represented as a function $S\left(1-\beta \sin\frac{\pi n}{2}\right)$, where $\beta << 1$ is a small parameter that characterizes the change in demand fluctuations.

Using the procedure that has been described above, order quantity $q^*{}_{opt}$ is sought as an asymptotic expansion in two small parameters ε and β, neglecting the terms ε^3, $\varepsilon^2\beta$ and ones of higher order:

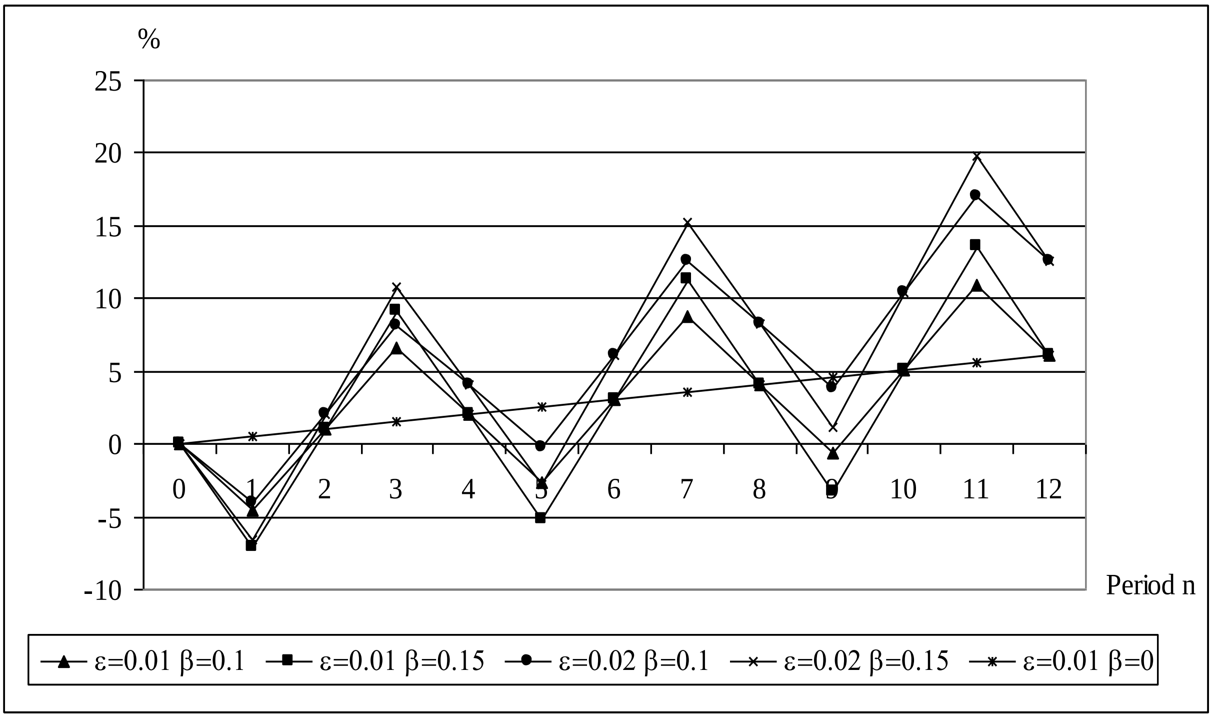
$$q^*{}_{opt} = (q_0 + \beta \cdot q_1) + (\tilde{q}_0 + \beta \cdot \tilde{q}_1) \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \ldots \quad (25)$$

The asymptotic formula for the "perturbed" order quantity takes the form:

$$q^*{}_{opt} = \sqrt{\frac{2C_0 S}{h}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon - \frac{1}{2}\beta \sin\frac{\pi n}{2} - \frac{n}{4}\varepsilon \cdot \beta \sin\frac{\pi n}{2} + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right). \quad (26)$$

Figure 4 demonstrates the percentage of "perturbed" order quantity deviation from the economic one provided gradual increase of per order cost and demand fluctuations.

Figure 4: "Perturbed" Order Quantity Deviation from the Economic Order Quantity Provided Gradual Increase of per Order Cost and Demand Fluctuations



Source: Calculated by the authors.

As we can see in Figure 4 the increasing order cost and fluctuations in demand cause order quantity fluctuations. Moreover, the more are the values of perturbation parameters ε and β the more significant is deviation from the economic order quantity (2). Figure 4 shows that in odd periods *n=3, 7, 11, ...* provided parameter ε is fixed, increase in amplitude of demand fluctuations leads to increase in order quantity. So, in period 3 in the case when cost per order and demand increase by 2 % ($\varepsilon = 0.02$) and 15 % ($\beta = 0.15$) respectively, the "perturbed" order quantity increases by 10.7 % in comparison with economic order quantity.

6. Conclusion

This study has proposed an asymptotic approach to Economic Order Quantity model. The approach is based on using the perturbation methods that make it possible to find a solution of the problem provided a small range of variation of system parameters.

The simple asymptotic formulas for the "perturbed" order quantity has been obtained when cost per order, storage cost and product demand change slightly. Despite the fact that the total costs that correspond to the "perturbed" order quantity do not differ significantly from those which correspond to the economic one, in scales of the total purchases of the machine-building enterprises cost reductions can be significant.

This study by proposing an asymptotic approach to Economic Order Quantity model will help to develop analytical logistic models of inventory management provided variable both per order and storage cost and fluctuations in demand for products. Modeling the nature of changes in demand and costs by using the asymptotic formulas, manager can make timely adjustments to purchasing organization at the enterprise.

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